8.1 Sequences

definition: list (ordered)

function, domain = ~~whole numbers~~ counting numbers

1.1 1.2 *Unsaved

Find the first six terms and the 100th term of $a_n = \frac{(-1)^n}{n^2+1}$

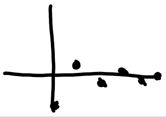
n: 1, 2, 3, 4, 5, 6, ..., 100

$-\frac{1}{2}, \frac{1}{5}, -\frac{1}{10}, \frac{1}{17}, -\frac{1}{26}, \frac{1}{37}, \dots \frac{1}{10{,}001}$

3 5 7 9 . . .

2 2 2

then graph the sequence.



converges to 0

$\lim_{n\to\infty} \frac{(-1)^n}{n^2+1} = 0$

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Arithmetic sequences — linear, common difference {add same value} d

definition and nth term expression:

$\boxed{a_n = a_1 + d(n-1)}$

recursive: $a_{n+1} = a_n + d$

n = 0 1 2 3 4 5 6 7 8 ... 100

Find ~~explicit and~~ recursive definitions for {-5,-2,1,4,7...} $-8, \dots, 10, 13, 16, \dots 292$

3 3 3 3 ...

$a_{n+1} = a_n + 3$
$a_1 = -5$

write next 3 terms, 100th term

$a_n = 3n - 8$ $\quad \cancel{-8} + 3\cdot 100$

$a_{100} = 3\cdot 100 - 8 = 292 \quad -5 + 3\cdot 99$

slope = 3 pt-slope (1,-5)

$a_n = 3(n-1) - 5$

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The fifth and ninth terms of an arithmetic sequence are 5 and -3, respectively. Find the first term and an explicit rule for the nth term.

1 2 3 4 5 6 7 8 9 (4)

13 11 9 7 5 - - - - -3

2 2 2 2 $\quad$ -8

$\frac{-8}{4} = -2$

$a_n = 13 + -2(n-1)$

$\frac{y_2-y_1}{x_2-x_1} \quad \frac{-3-5}{9-5} \quad \frac{-8}{4} = -2$

$a_n = a_1 + -2(n-1)$

$5 = a_1 + -2(5-1)$

$5 + 2\cdot 4 = a_1$

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Geometric Sequences (exponential) common ratio r

recursive and explicit definitions

nth term $a_n = a_1 \cdot r^{n-1}$

recursive $a_{n+1} = a_n \cdot r$ $\qquad \frac{a_{n+1}}{a_n} = r$

How much will you have if one penny is doubled 32 times?

$a_n = 2^n$ n starts at 0: 0 1 2 3 4 5 6 ... 32

1 2 4 8 16 32 64 ... 2^{31} 2^{32} (×2 ×2 ×2)

1 2 3 4 5 6 7 ... 32 33

n starts at 1 $a_n = 2^{n-1}$

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The second and fifth terms of a geometric sequence are 6 and -48 respectively. Find an explicit expression for the nth term.

1 2 3 4 5

$\_\ 6\ \underbrace{\_\ \_}_{3}\ -48$ $-8 = r^3$

$\frac{-48}{6} = -8$ $\sqrt[3]{-8} = -2 = r$

need $a_1 = \frac{6}{-2} = -3$ $a_n = -3(-2)^{n-1}$

$6 = a_1 r^1$ $a_1 = \frac{6}{r}$ $\frac{6}{r} = \frac{-48}{r^4}$

$-48 = a_1 r^4$ $a_1 = \frac{-48}{r^4}$ $r = -2$

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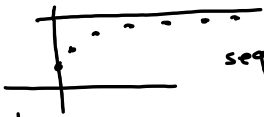
Infinite sequences

1.1 1.2 1.3 *Unsaved

Determine whether the sequence converges or diverges. If it converges, find its limit. Use symbolic and graphical methods.

$a_n = \frac{2n-1}{n}$



HA $y=2$

seq. converges to 2

$\lim_{n \to \infty} \frac{2n-1}{n} = \lim_{n\to\infty} \frac{2n}{n} = 2$

numerical $n = 100$ $\frac{199}{100} \approx 2$

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The sandwich theorem

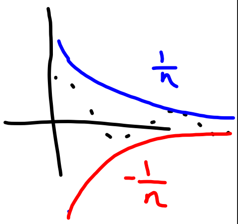
1.2 1.3 1.4 *Unsaved

Show the sequence $\{\frac{\cos n}{n}\}$ converges and find its limit.

converges to 0

$\frac{-1}{n} \le \frac{\cos(n)}{n} \le \frac{1}{n}$

$\lim_{n\to\infty}$ 0 0 0



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